CBCS SCHEME

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20SCN/SCS/SSE/SFC/LNI/SAM11

First Semester M.Tech. Degree Examination, July/August 2022
Mathematical Foundations of Computer Science

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Let $S = \{V_1, V_2, V_3, V_4\}$ be a basis for R^4, where $V_1 = (1, 1, 0, 0)$, $V_2 = (2, 0, 1, 0)$, $V_3 = (0, 1, 2, -1)$ and $V_4 = (0, 1, -1, 0)$. If $V = (1, 2, -6, 2)$, compute $[V]_S$. **(10 Marks)**

b. Show that the set $S = \{t^2 + 1, t - 1, 2t + 2\}$ is a basis for the vector space P_2. **(10 Marks)**

OR

2 a. Find the eigen values and eigen vectors of the matrix of transformation $T : R^2 \to R^3$ defined as $T(e_1) = (1, 2)$ and $T(e_2) = (3, 2)$. **(10 Marks)**

b. Find the matrix of linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (x + y, y + z)$ relative to the basis $B_1 = \{(1, 1, 1), (1, 0, 0), (1, 1, 0)\}$ and $B_2 = \{(1, 0), (0, 1)\}$. **(10 Marks)**

Module-2

3 a. Define an inner product space. For any vectors α, β in an inner product space V and any scalar C, prove that

(i) $\| C\alpha \| = C \| \alpha \|$ (ii) $\| \alpha \| > 0$ for $\alpha \neq 0$

(iii) $\| (\alpha/\beta) \| \leq \| \alpha \| + \| \beta \|$ (iv) $\| \alpha+\beta \| \leq \| \alpha \| + \| \beta \|$ **(10 Marks)**

b. Define orthogonal projection. Prove that an orthogonal set 'S' of non-zero vectors in $\mathbb{R}^n$ is linearly independent and hence forms the basis for the subspace spanned by the set S. **(10 Marks)**

OR

4 a. Find the QR factorization of matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ **(10 Marks)**

b. Find the least square solution of the system Ax = b for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$ and also determine the least square error in the solution of Ax = b. **(10 Marks)**

Module-3

5 a. Orthogonally diagonalize the matrix

$$\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$ **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Convert the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into the quadratic form with no cross product. **(10 Marks)**

OR

6 a. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraint $X^TX = 1$. **(10 Marks)**

b. Find the singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. **(10 Marks)**

Module-4

7 a. Find the correlation co-efficient and line of regression of y on x for the following data:

x	1	2	3	4	5
y	2	5	3	8	7

(10 Marks)

b. Fit a straight line for the following data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(10 Marks)

OR

8 a. Show that $\tan\theta = \left(\frac{(1-r)^2\theta}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_{x^2}\sigma_{y^2}}\right)$, where θ is the acute angle. Explain the significance of the formula when $r = 0$ and $r = \pm 1$. **(10 Marks)**

b. The two regression lines for the variables x and y are given by $x = 19.13 - 0.087y$ and $y = 11.64 - 0.50x$, find
(i) Mean of x and y (ii) Correlation coefficient of x and y. **(10 Marks)**

Module-5

9 a. A random variable X has the following probability mass function for various values of X:

X	0	1	2	3	4	5	6	7	8
P(X)	K	3k	5k	7k	9k	11k	13k	15k	17k

Solve : (i) Value of k
(ii) $P(X < 3)$, $P(X \geq 3)$, $P(0 < X \leq 5)$
(iii) Find cumulative distribution function.
(iv) What is the smallest value of X for which $P(X \leq x) > 0.5$ **(10 Marks)**

b. A certain injection administered to each of 12 patients resulted in the following increases of blood pressure:
5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4
Can it be concluded that the injection will be, in general, accompanied by an increase in BP? ($t_{0.05}(\nu = 11) = 2.201$) **(10 Marks)**

OR

10 a. A random variable X has the following probability density function :

$$P(X) = \begin{cases} ke^{-x}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Solve : (i) Value of constant k (ii) Mean (iii) Variance (iv) F(0.5) **(10 Marks)**

b. The following data give the number of aircraft accidents that occurred during the various days of a week.

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of Students	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week.

[Use $\chi^2_{0.05}(\nu = 5) = 11.07$] **(10 Marks)**

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